TOPICS IN QUANT TRADING

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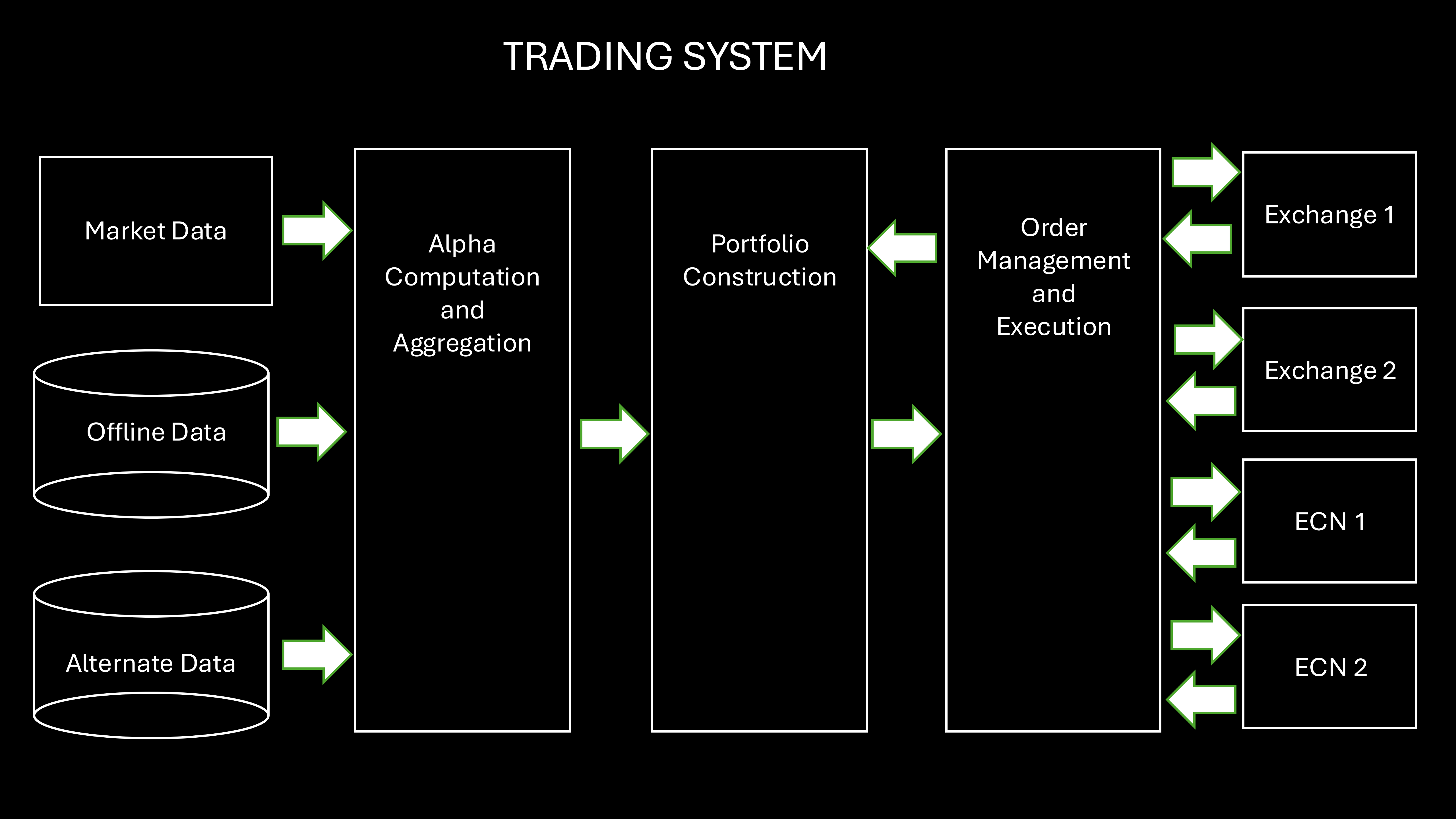
18 February 2025

Substack: `@MaverickQuant`


TRADING SYSTEM
Market Data
Offline Data
Alternate Data
Alpha Computation and Aggregation
Portfolio Construction
Order Management and Execution
Exchange 1
Exchange 2
ECN 1
ECN 2

1. Data Processing
2. Best Execution

1. Data Processing

Data Types:

- Time & Sales (T&S)
- Quotes: level 1 / 2 / 3
- Non-tradable data: indices, implied vols, ...
- Non-market data

Issues:

- Latency
- Outliers
- Reliability; real-time monitoring

1. Data Processing

Quotes vs Trades:

- Alpha computation / Execution
- Shadow price
- Alignment
- Cross-Validation and filtering
- Spread information; liquidity

Non-market and alternative data:

- Look-ahead bias is a major concern: missing timestamps, revisions, deletions
- Using an LLM / AI algo without knowing the exact training data set
- Survivorship bias (especially for equity markets)
- Time inconsistency
- Contamination with AI-generated content (e.g. X, Discord feeds)

Best exec. in the presence of drift, i.e. EXECUTION ALPHA**:**

$$\begin{cases} ds_t = \mu dt + \sigma dW_t \\ dq_t = -\lambda_t dt; \quad q_0 > 0, \quad q_T = 0 \\ dx_t = (s_t - \beta\lambda_t)\lambda_t dt \end{cases}$$

$$u(t,s,x,q) = \sup_{\lambda_t;\, q_T=0} \mathbb{E}\left(x_T + q_T s_T\right)$$

$$(HJB) \begin{cases} u_t + \mu u_s + \frac{\sigma^2}{2} u_{ss} + \sup_{\lambda} \{(s-\beta\lambda)\lambda u_x - \lambda u_q\} = 0 \\ \\ u(t=T,s,x,q) = x \ \text{ if } \ q=0, \quad -\infty \ \text{ otherwise} \end{cases}$$

$$u(t,s,x,q) = x + sq - h(t,q)$$

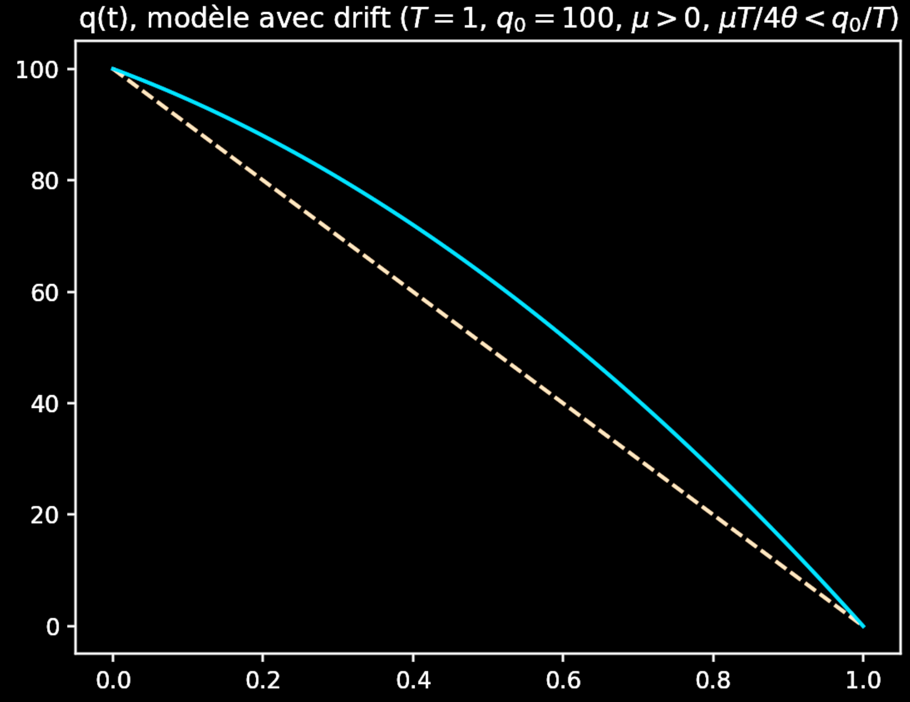
$$-h_t + \mu q + \sup_{\lambda}\left\{-\beta\lambda^2 + \lambda h_q\right\} = 0$$

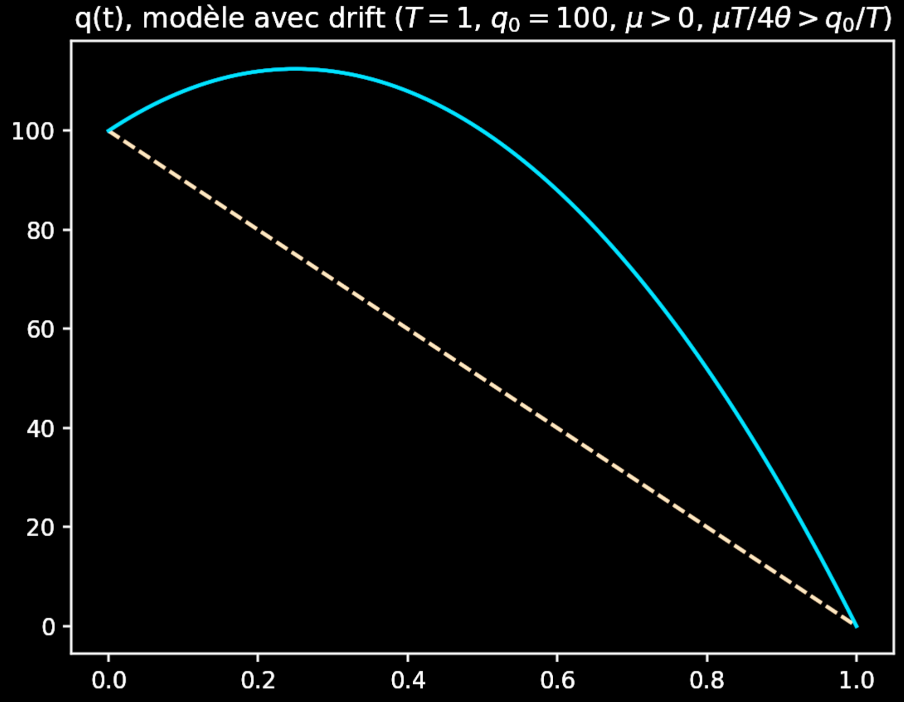
$$-h_t + \mu q + \frac{1}{4\beta}h_q^2 = 0$$

$$-\frac{dq}{dt} = \lambda^* = \frac{1}{2\beta}h_q(t,q(t))$$

$$-\frac{d^2q}{dt^2} = \frac{1}{2\beta}h_{qt} - \frac{1}{4\beta^2}h_q h_{qq} = \frac{\mu}{2\beta}; \qquad q(0) = q_0, \quad q(T) = 0$$

$$\boxed{q(t) = (T-t)\left(\frac{q_0}{T} + \frac{\mu}{4\beta}t\right)}$$


q(t), modèle avec drift ($T = 1$, $q_0 = 100$, $\mu > 0$, $\mu T/4\theta < q_0/T$)
100
80
60
40
20
0
0.0
0.2
0.4
0.6
0.8
1.0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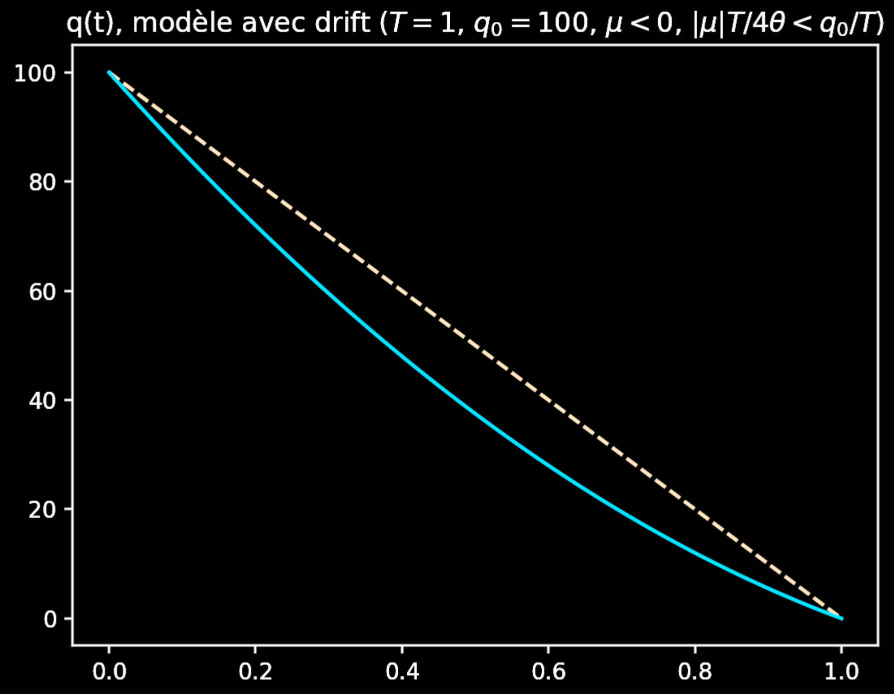
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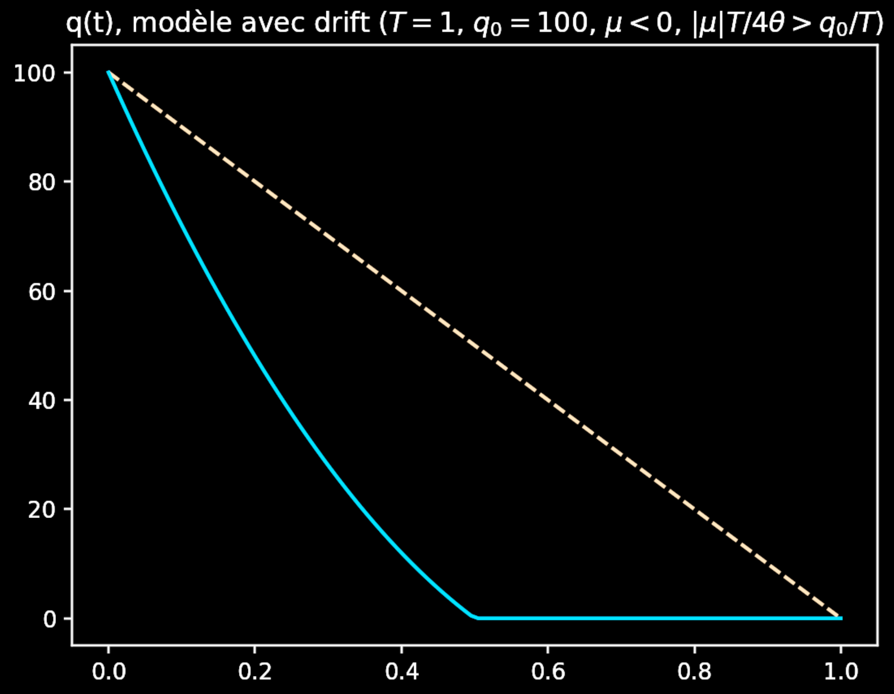
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Best exec.: passive or aggressive?

Choice between marketable order at price $s - \delta_0$ ($s =$ mid, δ_0 half-spread), or limit sell at $s + \delta$ ($\delta > 0$)

Assume order flow intensity hitting the offer side follows

$$A \frac{1}{(\delta + \delta_0)^k} \quad (A > 0,\ k > 1)$$

Our utility function is

$$u(s, x, q, t) = \sup_{\delta} \mathbb{E}\left(x_T + q_T(s_T - \delta_0)\right)$$

$$\begin{cases} \max\left\{u_t + \frac{\sigma^2}{2}u_{ss} + A\sup_{\delta>0}\frac{1}{(\delta+\delta_0)^k}\left(u(s,x+(s+\delta)q_0,q-q_0,t) - u(s,x,q,t)\right);\right. \\ \left. u(s,x+(s-\delta_0)q_0,q-q_0,t) - u(s,x,q,t)\right\} = 0 \\ u(s,x,q,t=T) = x + q(s-\delta_0) \\ u(s,x,q=0,t) = x. \end{cases}$$

$$u(s,x,q,t) = x + q(s-\delta_0) + h(q,\tau), \quad \tau = T - t$$

2. Best Execution

$$\begin{cases} \min\left\{h_\tau - Aq_0 \sup_\delta \frac{1}{(\delta+\delta_0)^k}(\delta+\delta_0-h_q)\,;\; h_q\right\} = 0 \\ h(0,\tau) = 0; \quad h(q,0) = 0, \end{cases}$$

$$\delta^* = \left(-\delta_0 + \frac{k}{k-1} hq\right)_+, \quad (z_+ = \max(z,0))$$

$$\begin{cases} \min\{h_\tau - F(h_q), h_q\} = 0 \\ h(0,\tau) = h(q,0) = 0, \end{cases} \qquad F(v) = \begin{cases} \frac{Aq_0}{k}\left(1-\frac{1}{k}\right)^{k-1} v^{1-k} & \text{if } v \geq \left(1-\frac{1}{k}\right)\delta_0 \\ Aq_0(\delta_0^{1-k} - \delta_0^{-k} v) & \text{otherwise.} \end{cases}$$

$$\begin{cases} \min\left\{h_\tau - F(h_q), h_q\right\} = 0 \\ \\ h(0,\tau) = h(q,0) = 0, \end{cases}$$

Hopf-Lax-Oleinik formula

$$h(q,\tau) = \tau F^*\left(\frac{q}{\tau}\right), \qquad F^*(z) = \inf_{v>0}\left(zv + F(v)\right) \qquad (z>0)$$

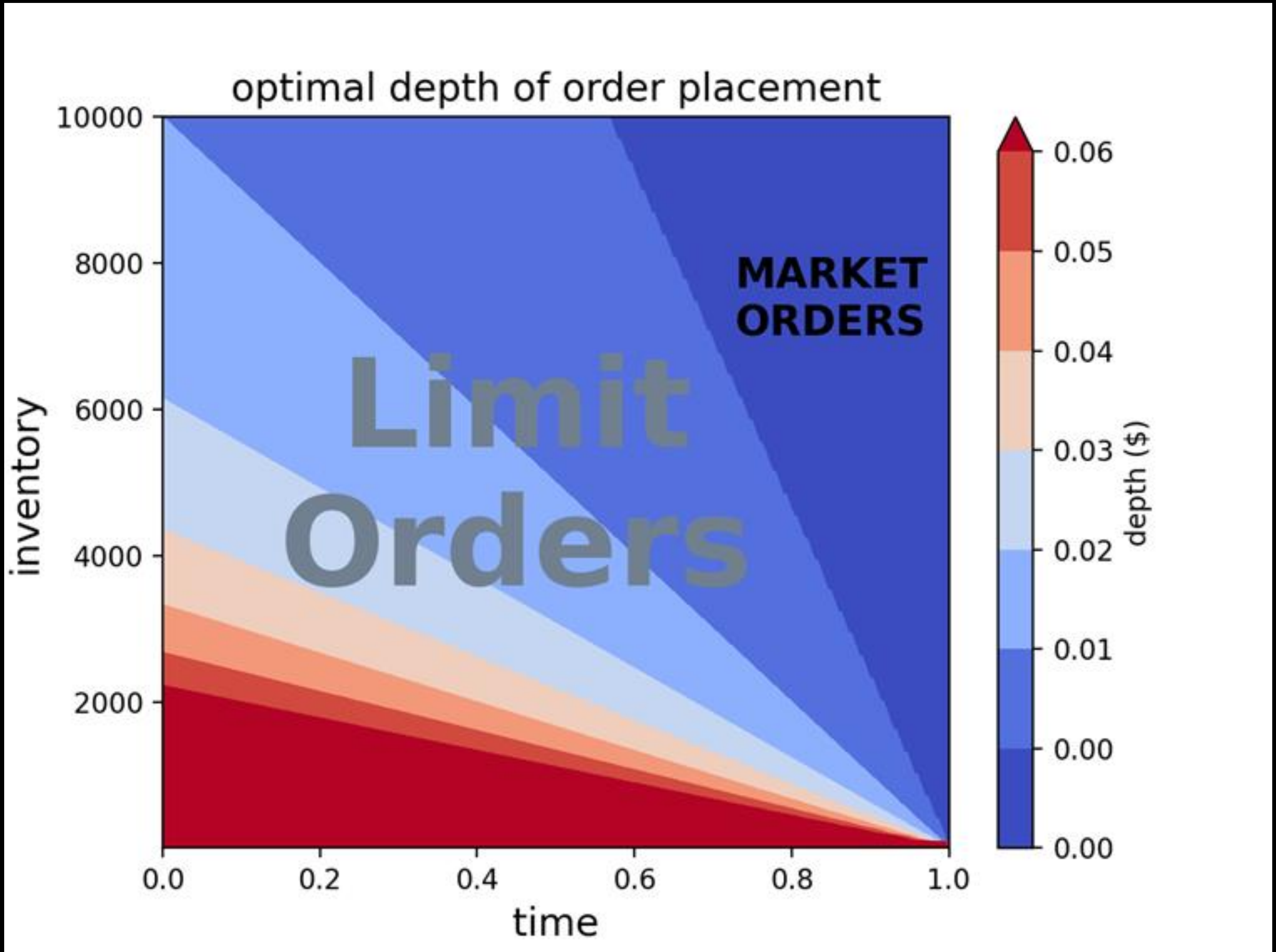
$$h(q,\tau) = \begin{cases} (Aq_0)^{1/k}\,\tau^{1/k} q^{1-1/k} & \text{if } \dfrac{q}{\tau} < Aq_0\delta_0^{-k} \qquad (i) \\ \\ Aq_0\delta_0^{1-k}\tau & \text{otherwise.} \qquad (ii) \end{cases}$$

Continuation region

$$\delta^* = (Aq_0)^{1/k} \left(\frac{\tau}{q}\right)^{1/k} - \delta_0 > 0, \quad \text{if} \ \frac{q}{\tau} < Aq_0\delta_0^{-k}$$

Stopping region

$$\frac{q}{\tau} \geq Aq_0\delta_0^{-k}$$


optimal depth of order placement
10000
8000
6000
4000
2000
inventory
0.0
0.2
0.4
0.6
0.8
1.0
time
Limit Orders
MARKET ORDERS
0.06
0.05
0.04
0.03
0.02
0.01
0.00
0.00
depth ($)

Expected returns $e = (e_1, \cdots, e_n)$, current position $p^0 = (p_1^0, \cdots, p_n^0)$

Transaction costs $\alpha = (\alpha_1, \cdots, \alpha_n)$, risk aversion $\lambda > 0$, covariance matrix Ω

$$\sup_{p \in \mathbb{R}^n} \left\{ p^T e - \frac{\lambda}{2} p^T \Omega p - \sum_{k=1}^{n} \alpha_k |p_k - p_k^0| \right\} \longrightarrow p^*$$

$+$ constraints...

$\bar{p} = \lambda^{-1} \Omega^{-1} e$, $\quad \Omega = \sum_{k=1}^{n} \mu_k u_k u_k^T$, $\mu_1 \geq \cdots \geq \mu_n$ $\quad$ condition number: $\mu_1 / \mu_n \longrightarrow$ eigenvalues rectification

Dual / Legendre Formulation

$$\phi_1(p) = \frac{\lambda}{2} p^T \Omega p, \quad \phi_2(p) = \sum_{k=1}^{n} \alpha_k |p_k - p_k^0|, \quad \phi = \phi_1 + \phi_2$$

$$\phi^*(e) \equiv \sup_p \left\{ p^T e - \phi(p) \right\} \longrightarrow \text{hence portfolio optimization as Legendre transform}$$

$$\phi^* = (\phi_1 + \phi_2)^* = \phi_1^* *_{inf} \phi_2^*$$

$$\phi_1^*(\xi) = \frac{\lambda^{-1}}{2} \xi^T \Omega^{-1} \xi$$

$$\phi_2^*(\xi) = p_0^T \xi + \mathbf{1}^{\infty}_{|\xi|_i < \alpha_i}(\xi), \quad \mathbf{1}^{\infty}_{|\xi|_i < \alpha_i}(\xi) = 0 \text{ if } |\xi|_i < \alpha_i \forall i; \; +\infty \text{ otherwise}$$

$$\phi^*(e) = \phi_1^* *_{inf} \phi_2^*(e) = \inf_{\eta} \left\{ \frac{\lambda^{-1}}{2}(e-\eta)^T \Omega^{-1}(e-\eta) + \left(p^0\right)^T \eta + \mathbf{1}^{\infty}_{|\eta|_i<\alpha_i}(\eta) \right\}$$

$$= \boxed{\inf_{|\eta|_i<\alpha_i} \left\{ \left(p^0\right)^T \eta + \frac{\lambda^{-1}}{2}(e-\eta)^T \Omega^{-1}(e-\eta) \right\}}$$

$$D\phi(p^*(e)) = e$$

$$\boxed{p^*(e) = D\left(\phi^*\right)(e)}$$

Robust Optimization

$$\bar{U}_\delta(\hat{e}) = \left\{ e \,|\, (e - \hat{e})^T \Sigma^{-1} (e - \hat{e}) \leq \delta^2 \right\}$$

$$p^* = \arg \max_{p^T \Omega p \leq R^2} \left\{ \min_{e \in \bar{U}_\delta(\hat{e})} \left(e^T p - \frac{\lambda}{2} p^T \Omega p \right) \right\}$$

It can be shown (using Lagrangian) that:

$$p^* = \arg \max_{p^T \Omega p \leq R^2} \left\{ \hat{e}^T p - \frac{\lambda}{2} p^T \Omega p - \delta \sqrt{p^T \Sigma p} \right\}$$

Different Types of Risk

- Market Risk: volatility estimation, VaR, tail risk
- Model Risk: mispricing
- Operational Risk

THE WALL STREET JOURNAL.

Loss Swamps Trading Firm

Knight Capital Searches for Partner as Tab for Computer Glitch Hits $440 Million

By Jenny Strasburg And Jacob Bunge

Updated Aug. 2, 2012 8:10 pm ET

Gift unlocked article

Knight Capital Group Inc. scrambled Thursday to shore itself up and reassure panicked customers after disclosing a stunning $440 million loss from a computer-trading glitch.

Knight officials blamed software installed earlier this week for causing the brokerage firm to enter millions of faulty trades in less than an hour on Wednesday morning. The orders roiled trading in almost 150 stocks and left Knight holding losing positions in many shares at the end of Wednesday's trading session.

References

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